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THE RATIONALE OF ARITHMETICAL TEACHING,

EXEMPLIFIED IN A FULL EXPOSITION OF THE

PRINCIPLES OF NUMERATION

AND THE

FOUR ELEMENTARY RULES;

WITH

Remarks on Teaching Arithmetic.

BY

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THE

RATIONALE

OF

ARITHMETICAL TEACHING.

INTRODUCTORY REMARKS.

1. In the following section on Numeration, it is shown in order how to name and represent all numbers, from the simplest to the largest. It would not be proper to pursue the same course in teaching. The plan followed may be something like the following:—

2. Let the children be first exercised mentally, for some time, in the four fundamental rules with simple *concrete* numbers; notions of number being thus gradually acquired, the symbols for the first nine numbers may be taught; after clear ideas of abstract numbers have been acquired by calculations with these, counting by tens may be proceeded with, and the nature of local value and the use of the cipher taught; afterwards, counting by hundreds, thousands, &c., each stage being thoroughly mastered before the next is entered upon: an anxiety to get quickly through defeats itself in teaching the elements of any subject. Children have been very much amused with the following way of teaching the decimal scale, at the same time that they rapidly acquired clear notions of counting by tens. A child is taken

from the class, who stands before it and holds up a finger for every object to which the master points (the intention being to count the number of objects in the room); when the eleventh object, to which the master rapidly passes, is reached, the boy calls on him to stop, he has not eleven fingers. It is then determined that another boy shall stand out to hold up a finger whenever the first is ready to begin again: this process may be carried to any length. At the end columns are ruled, and the number of fingers each child has held up is registered in order.* It is then shown how the columns may be dispensed with; the necessity for the cipher will appear when the lines are removed in cases where there are none of any kind of unit. By such methods the children are led *naturally* to the decimal scale; they, as it were, discover it for themselves. It is very important to teach Numeration thoroughly, for the processes in Addition, Subtraction, Multiplication, Division, and the rules relating to Decimals are directly founded on its principles. The proposed introduction of a Decimal Coinage gives additional force to this remark.

3. Children, in general, are made to begin Slate-arithmetic too soon; they frequently do so when they hardly know the symbols for the first nine numbers. They have, in consequence, to encounter at once a multitude of difficulties, each of which should be conquered alone; such as the strangeness of the whole subject, the labour of adding, their unfamiliarity with the figures, and awkwardness in making them, and the process of carrying. They should have nothing to do with slates until they are *at least* sufficiently versed in mental calculation to perform promptly all the mental processes required in the fundamental rules. These rules are supposed to be explained in the following pages to those who have undergone this previous training.

* This arithmetical game, as it may be called, is simply the illustration used in Art. 9. of Professor De Morgan's Arithmetic, carried into practice.

4. In entering on any new rule, a question should first be proposed by means of which the requisite definitions may be brought in; if a new symbol of operation is necessary, this should next be explained; then, the principles employed in establishing the rule are to be illustrated by a sufficient number of examples, — these illustrations always *preceding* the formal statement of the principle, which must be *inferred* from them; after this, many examples of the kind required, from the simplest to the most difficult, must be done by the help of the principles previously elucidated; and, lastly, the pupils should be guided by varied forms of interrogation in deriving the rules for themselves by an induction from these examples. It is especially to be remembered, that in this process of demonstration the pupils must be *told* as little as possible: the intuitive or Socratic method of teaching is the one to be pursued here. In order to further the education of that important faculty, the memory, by training it to habits of retention and precision, every principle or rule, as soon as it is thoroughly understood, should be expressed in simple and precise language, and required to be learnt by heart and repeated without deviation. In attending to principles so as to develop the faculties of abstraction, attention, and reasoning, the teacher must take care to exercise his pupils sufficiently to render easy the application of these principles. It would be a great fault to enter so minutely on processes of demonstration as to neglect *practice* in a subject which bears so intimately on the concerns of every-day life as Arithmetic. *Facility*, *correctness*, and *neatness* in working, should then be looked after. All such words as *Addition*, *Sum*, *Excess*, *Product*, *Interest*, *Rate*, &c. should be explained, and precise explanations required in return, as it conduces much to clearness of thinking when a habit is acquired of requiring and giving correct definitions.

5. It is frequently urged against teaching Arithmetic demonstratively, that no time can be found for it, and that it in no way conduces to accuracy and quickness of working.

In order to remove these objections, and to show the necessity of making time for such teaching, it is sufficient to state what are the objects proposed to be attained by it. There are two ways in which, generally, we may look at any subject of instruction; we may view it as an *end* in itself, on account of its utility or the pleasure an acquaintance with it affords, or as a *means* for training some faculty or faculties, that is, as an instrument of education. It depends on what we seek to accomplish, to which aspect most prominence is given; but in every sound system of primary instruction, while the *subjects* are largely decided by the former, the *methods* will be entirely determined by the latter. Thus, Arithmetic, with which only we are concerned, may be taught simply in its connexion with the business of life; or, in addition to this, as a means of forming the attention and judgment, and developing the reasoning powers. In the former case, the teaching will be confined to the mechanism of calculation; in the latter, reasons will be given with rules, or rather, as formerly explained, the children will be led, by careful training, to prove the rules for themselves. Of the two courses here indicated, the latter is certainly the preferable one; for, although it is quite true that the most expert calculators are frequently those who are ignorant of abstract principles, and that long and laborious drilling is the only way to make one sure and quick in the use of numbers, yet it is more important that a boy should have a well-trained mind, than that he should be an expert calculator. Besides, attention to one part of the subject does not imply neglect of the other. In the education of those whose school-life is of limited duration, it is especially necessary that some elementary subjects be used as disciplinary agents, for it is to such subjects only that they have access, and if they are to be trained to sound intellectual habits at all, it must be by means of judicious instruction in them. It is then, as a mental gymnastic, as a means of imparting the art of thinking (if the expression

may be allowed), and producing certain invaluable mental aptitudes, that Arithmetic is taught in the careful way above described, so that it may fill, in its measure, the same place in primary, that the Higher Mathematics do in superior education.

NUMERATION AND NOTATION.

6. If we wish to express that there is a single object of any kind, instead of writing *one*, for the sake of shortness we use the mark or *figure* . . . 1

If to one thing we add another of the same kind, we shall have *two* things. Thus, *one* apple and *one* apple make *two* apples. To express this *number* we use the figure 2

If to *two* things we join *one* thing of the same kind, we shall have *three* things. Thus, *two* boys and *one* boy are *three* boys. This number is expressed by the figure 3

If to *three* we add *one*, we shall have *four* . . 4

Four and *one* are *five* 5

Five and *one* are *six* 6

Six and *one* are *seven* 7

Seven and *one* are *eight* 8

Eight and *one* are *nine* 9

If to the number *nine* we add *one*, we shall have the number *ten*.

7. In this way we may form as many numbers as we please, but it is plain that we cannot give fresh names to all of them; if we were to do so, we would at last be stopped by the impossibility of remembering these names, and thus we would not be able to reckon with very large numbers. A shepherd, for example, would not be able to state the number of sheep in a large flock. I am going to explain how all

numbers may be named by means of very few words in addition to *one*, *two*, . . . *nine*.

8. Suppose there are before us many objects, say marbles, which we wish to count. In order that we may be able easily to discover how far we had gone, if we happen to lose count, or to resume our task where we left off, after having dropped it for some time, we take ten out and put them aside by themselves (why this number is taken rather than any other will be noticed by-and-by); we take ten more out, and put them aside; ten more, and put them aside; and so on. A set of heaps is thus formed, each containing ten things. When there is *one* heap, we have counted *ten*; when there are *two* heaps, we have counted *ten twice*; when there are *three* heaps, we have counted *ten three times over*; and so on. The names of the numbers thus formed are shortened as follows:—

Two tens	into	*Twenty*,
Three tens	„	*Thirty*,
Four tens	„	*Forty*,
Five tens	„	*Fifty*,
Six tens	„	*Sixty*,
Seven tens	„	*Seventy*,
Eight tens	„	*Eighty*,
Nine tens	„	*Ninety*.

When we have taken out one heap, before the second heap is formed we shall have successively — ten marbles and one marble more, ten marbles and two marbles more, ten and three more, &c.; these numbers are named thus:—

Ten and one are *eleven*,
Ten and two are *twelve*,
Ten and three are *thirteen*,
Ten and four are *fourteen*,
Ten and five are *fifteen*,
Ten and six are *sixteen*.

Ten and seven are *seventeen*,
Ten and eight are *eighteen*,
Ten and nine are *nineteen*.

In the same way we can add one after the other of the first nine numbers to two tens or twenty, next to thirty, next to forty, &c., getting *twenty and one*, which is shortened into *twenty-one; twenty and two*, which is shortened into *twenty-two*, &c. Hence we have the following numbers:—

Twenty-one, twenty-two, ... twenty-nine;
Thirty-one, thirty-two, ... thirty-nine;
&c. &c. &c.
Ninety-one, ninety-two . . . . ninety-nine.

Making use of the figures which have been given above, we may write the numbers just formed in this way: 1 *ten*, 1 *ten* 1 *one* (that is, 1 *ten* and 1 *one*) or eleven, 1 *ten* 3 *ones* or thirteen, &c., 2 *tens* or twenty, 2 *tens* 1 *one* or twenty-one, &c., 9 *tens* 9 *nines* or ninety-nine; but instead of writing 1 *ten* 1 *one*, it is plain that we may write 11, if we remember that after the figure to the left the word *ten* is understood, and after the other figure the word *one*. In the same way 12 will be read as 1 *ten* 2 *ones* or twelve, 37 as 3 *tens* 7 *ones* or thirty-seven, 82 as eighty-two, and so on. But if we omit the word in—1 *ten*, 2 *tens*, 3 *tens*, &c., these will be taken for 1 *one*, 2 *ones*, 3 *ones*, &c., respectively. How shall we manage in this case? In addition to the nine figures already given, another character, viz. 0, is used, to represent *nothing:* it is called *nought*, *cipher*, or *zero*. Hence we may write 1 *ten* as 1 *ten* 0 *ones* (that is, 1 *ten* and nothing more), 2 *tens* as 2 *tens* 0 *ones*, 3 *tens* as 3 *tens* 0 *ones*, &c.; and now if we omit the words, and leave 10, 20, 30, &c., no mistake can arise, for we see at once, as before, that there are, in each case, so many tens and no ones.

Thus, then, *if we wish to express a number consisting of ones and tens, we place the figure stating the number of*

tens, or the tens' figure, to the left of the ones' figure; and when there are no ones we mark their place by a cipher; on the other hand, if a number be expressed by two figures, the figure to the right denotes the number of ones, and the figure before it to the left, the number of tens.

9. Let the process described at the beginning of the last article be carried on until there are left behind not so many as ten marbles. We have now a certain number of tens, and a number of ones not more than nine. Our next task is to count the number of heaps of ten. If this number does not exceed nine, the total number of marbles will be named and represented as explained in Art. 8. Suppose, however, that there are more than nine heaps; in order to count these we do with them what we did with the single marbles; that is, we take ten heaps and put them together by themselves, ten more and put them together, ten more and put them together, and so on, until there are left behind not so many as ten heaps. A set of heaps is thus formed, each of which contains *ten marbles ten times repeated.* An assemblage of ten tens is called a *hundred;* so that each new heap contains one hundred marbles, two such heaps contain two hundred, &c. When the process just described has been carried on as far as possible, we shall have, in general, a set of heaps each containing a *hundred,* another set of not more than nine each containing *ten,* and a number of *single* marbles not more than nine; that is, the total number of marbles will be expressed in hundreds, tens, and ones.

We count, then, by hundreds as by tens and by ones; thus, we may have one hundred, two hundreds, ... nine hundreds; and we can insert between one hundred and two hundred, between two hundred and three hundred, &c., the first ninety-nine numbers. The numbers thus formed are represented according to the principle followed in representing numbers made up of ones and tens. Hence we have: one hundred, or 1 *hundred* 0 *ten* 0 *ones,* 100; two hundred, 200; three hundred, 300; nine hundred, 900; one hundred

and one, or 1 *hund.* 0 *tens* 1 *one*, 101; one hundred and two, 102; one hundred and nine, 109; one hundred and ten, or 1 *hund.* 1 *ten* 0 *ones*, 110; one hundred and eleven, or 1 *hund.* 1 *ten* 1 *one* (that is, 1 *hund.* and 1 *ten* and 1 *one*), 111; one hundred and twelve, 112; one hundred and thirteen, 113; one hundred and twenty, or 1 *hund.* 2 *tens* 0 *ones*, 120; one hundred and twenty-one, 121; one hundred and twenty-five, 125; one hundred and thirty, 130; one hundred and seventy-seven, 177; two hundred and five, 205; two hundred and fifty, 250; seven hundred and twenty-four, 724; nine hundred and ninety-nine, or 9 *hund.* 9 *tens* 9 *ones*, 999.

From the above it follows that *as the ones' figure takes the first place in passing from right to left, and the tens' figure the second place, so the hundreds' figure takes the third place.*

10. If we carry on the process described in the last two articles one step further, heaps containing *ten hundred* marbles each will be formed. An assemblage of ten hundreds is called a *thousand.* Thus, in an army a company being supposed to contain a hundred men, and a regiment ten companies, a regiment would be said to consist of a thousand men, and we could then count the men, not by companies, but by regiments. We would say that the army consists of one, two, three, &c. regiments, that is, of one thousand, or two thousand, or three thousand, &c. men.

In order to count from one thousand to two thousand, we add successively to one thousand the first nine hundred and ninety-nine numbers. And we can do the same between two thousand and three thousand, between three thousand and four thousand, &c.

If we form in this way the number one thousand five hundred and thirty-four, or 1 *thous.* 5 *hund.* 3 *tens* 4 *ones*, by omitting the words *thousand, hundred, tens, ones*, it may be expressed by 1534, which is easily understood if it is remembered what words are supposed to come after the figures

1, 5, 3, 4, respectively. But in 1 *thous.*, if we leave out the word *thousand*, the figure 1, being alone, will represent 1 *one*. If, however, we write 1 *thousand* in the form 1 *thous.* 0 *hund.* 0 *tens* 0 *ones*, after making the omissions we have 1000, the meaning of which cannot be mistaken, and in which the figure 1, standing for a thousand, occupies the fourth place as in the former number. And so, if we have six thousand and thirty-eight, or 6 *thous.* 3 *tens* 8 *ones*, we see that it would not be proper to leave out the words after the figures, for we should then have 638, or six hundred and thirty-eight; but if we supply the wanting place of hundreds and write 6 *thous.* 0 *hund.* 3 *tens* 8 *ones*, we may do so, for the result 6038, read as before, expresses the number correctly. In the same way the following numbers are written: one thousand and one, 1001; one thousand and two, 1002; one thousand and ten, 1010; one thousand and eleven, 1011; one thousand and twenty, 1020; two thousand, 2000; two thousand and seven, 2007; two thousand and seventy, 2070; two thousand seven hundred, 2700; five thousand and thirty-seven, 5037; nine thousand nine hundred and ninety-nine, 9999.

11. We count by thousands, tens of thousands, and hundreds of thousands, as by ones, tens of ones, and hundreds of ones; and just as

	One thousand is represented by . .	1 000,
So	Ten thousand	10 000,
	Twenty thousand	20 000,
	Thirty thousand	30 000,
	&c. &c.	&c.
	One hundred thousand . . .	100 000,
	Two hundred thousand . . .	200 000,
	Three hundred thousand . . .	300 000,
	&c. &c.	&c.

In the same way, forty-five thousand eight hundred and six, or 45 *thous.* 8 *hund.* 0 *tens* 6 *ones*, is 45806; two hundred

and seventy-eight thousand three hundred and nineteen is 278319; eight hundred and thirty thousand and four, or 830 *thous.* 0 *hund.* 0 *tens* 4 *ones*, is 830004.

It will be seen from all this that *as ones, tens of ones, and hundreds of ones occupy the first, second, and third places respectively, in passing from right to left; so thousands, tens of thousands, and hundreds of thousands occupy the next three places, that is, the fourth, fifth, and sixth.*

12. Ten hundred thousand, that is, a thousand thousand, form a *million.*

We count by millions, tens of millions, hundreds of millions as by ones, tens of ones, hundreds of ones, and by thousands, tens of thousands, hundreds of thousands; and just as

One thousand	is expressed by . .	1 000,
So One million,	or *one thousand* thousand	1 000 000,
Two million,	or *two thousand* thousand	2 000 000,
Three million	. . . . .	3 000 000,
&c.	&c.	&c.,
Ten million .	. . . . .	10 000 000,
Twenty million	. . . . .	20 000 000,
&c.	&c.	&c.,
One hundred million	. . . .	100 000 000,
Two hundred million	. . . .	200 000 000,
Three hundred million	. . . .	300 000 000,
&c.	&c.	&c.

All the numbers between one million and two million, between two million and three million, &c., can be formed by adding successively, to one million, to two million, &c., the first nine hundred and ninety-nine thousand nine hundred and ninety-nine numbers.

Suppose we arrive, in this way, at the number two hundred and thirty-seven million four hundred and nineteen thousand six hundred and eighty-four; to represent this in figures we have only to put it in the form 237 *mil.* 419

thous. 684 *ones*, and miss out the words *million*, &c., getting 237 419 684, which is easily read if we remember the omissions. But nineteen million twenty-six thousand and nine, or 19 *mil.* 26 *thous.* 9 *ones*, would give, in the same way, 19269, which, according to what has been explained before, stands for nineteen thousand two hundred and sixty-nine; if, however, we write the number in question as 19 *mil.* 026 *thous.* 009 *ones*, putting 026 for 26 to show that there are no hundreds of thousands, and 009 to show that there are no hundreds and no tens of ones, we have by making the omissions, 19 026 009, which, read as before, gives the number correctly. It is not necessary to put 019 for 19, because there are no figures to the left of 19 to be kept in their proper places.

From all that has been said it follows that *when any number is expressed in figures, the first three figures passing from right to left stand for ones, tens of ones, hundreds of ones; the next three for thousands, tens of thousands, hundreds of thousands; and the next three for millions, tens of millions, hundreds of millions; the place of each wanting denomination being supplied by a cipher, unless there be no more figures to the left.*

13. It is thus easy to read any number expressed in figures. Suppose we have 8620173; by dividing this, as far as we can, into periods of three figures each, beginning at the right hand, thus, 8,620,173, we see at once from what has just been said that it represents 8 *mil.* 620 *thous.* 173 *ones*, or eight million six hundred and twenty thousand one hundred and seventy-three. In the same way 200106 or 200,106 is two hundred thousand one hundred and six.

14. We very seldom meet with numbers greater than those given above, but it is easily seen that by following the same method as that hitherto pursued they may be named by employing a moderate number of words; and

that all numbers may be represented by means of the nine digits and the cipher.

15. NUMERATION *is the art of forming numbers, and of naming them in few words;* NOTATION, *of representing them by means of certain characters or figures.*

It will now be apparent that our system of Numeration consists, first, in arranging whatever objects may have to be counted in groups of such kind that ten members of any group form one of the next, and then in stating how many there are of each; and our system of Notation in using certain arbitrary characters or figures to denote the number of individuals in each group, and writing them one after the other in a fixed order, so that it may be known of which group any figure represents the number from its position with respect to the other figures: also, since there cannot be more than nine in any group (because when there are ten they are formed into one of the next), that in the former, the only names wanted are those of the first nine numbers and of the successive groups; in the latter, the only figures wanted are the nine digits, and, to denote the absence of any group, the cipher. It is to be observed that in naming the groups fresh names, that is, names not made up of others, are used only at intervals; thus, after a thousand come, successively, a ten-thousand, a hundred-thousand, a *million*, &c.

This method of naming and representing numbers is called the *decimal system*, from the Latin word *decem*, which means *ten*. Ten is said to be the *base of the system*.

It is clear that any number may be taken for base. Thus, in order to count a multitude of objects, suppose we take *seven* out and put them aside, seven more and put them aside, and so on until there are left behind not so many as seven. We have now a certain number of *sevens*, and a number of *ones* not more than six. Of the sevens, again, if there be more than six of them, seven may be taken to form a new set, seven more to form another set of the same kind, and so on until there are left behind not so many as seven

sevens. We have now a certain number of *seven-sevens*, not more than six *sevens*, and not more than six *ones*. By proceeding in this way we shall separate the collection of objects into groups of such kind that seven individuals of any group form one of the next. If this system were in use, therefore, names would only be needed for the first six numbers and for the successive groups; that is, in addition to *one*, *two*, *three*, *four*, *five*, *six*, *seven*, we should only want, for convenience sake, words to denote *seven-sevens*, &c., just as we now use *hundred*, &c., for *ten-tens*, &c.; and all numbers could be represented by means of six digits and a cipher. Thus, 54 would represent 5 *sevens* and 4 *ones*; 10, 1 *seven*; 100, 1 *seven-sevens*; 235, 2 *seven-sevens* 3 *sevens* 5 *ones*.

In the same way we could count by *sixes*, or by *elevens*, or by *fives*, &c., in every case the same end being gained of naming very large numbers by means of few words and of representing them by a limited set of figures.*

These considerations suggest the question, What caused the adoption of the decimal system? In all probability, the natural practice of reckoning on the fingers, which inevitably leads to counting by tens. So that if man had been gifted with twelve fingers instead of ten, our system of Numeration and Notation would have been *duodecimal*, that is, we should have reckoned by *twelves*; and the majority of those who learn arithmetic would never have thought of the possibility of any other method, just as we are now disposed to

* "The advantages of this resolution of numbers [into classes] are not confined to the expression of large numbers by few words, which are easily remembered; for we thus become familiar with the superior units, such as ten, a hundred, a thousand, as well from frequent repetition as from our knowledge of their relation to each other and to unity; and we are thus enabled to form clear and distinct conceptions of large numbers, whose composition we discover, in the words by which they are expressed, or in the symbols by which they are represented."—Dr. PEACOCK's *Arithmetic*, in *Ency. Met.*, Art. 6.

forget that our present plan is purely artificial*, and to regard it as having existence in the very nature of the case. Attentively considered, the device by which numbers, however large, are easily named, represented, and conceived of, must appear a crowning instance of man's ingenuity.

16. Two *men* and three *men* are five *men*, 2 *ozs.* and 3 *ozs.* are 5 *ozs.*, 2 *pens* and 3 *pens* are 5 *pens*, &c.; generally, 2 *things* and 3 *things of the same kind* are 5 *things of that kind*, whatever it may be; this is expressed briefly by saying that 2 and 3 are 5. Such numbers as 2 *oz.*, 3 *oz.*, 2 *men*, &c., are called *concrete* numbers; such numbers as 2 and 3 are called *abstract* numbers.

A concrete number, then, *is that of which the denomination is stated; an abstract number is that of which the denomination is not stated.*

ADDITION.

17. There are 12 boys in the first class, 17 in the second, 13 in the third, and 24 in the fourth, how many boys are there altogether in the school? This number, whatever it may be, is called the *sum* of 12, 17, 13, 24; in finding it we are said to *add* 12, 17, 13, and 24 together, and the process by which we find it is called ADDITION.

Addition, then, *is an operation by which that number is found which is as large as two or more numbers put together; the result of this operation is called the sum or total.*

To show that several numbers have to be added together,

* When considered, however, in reference to the natural practice which probably led to it, the decimal system may be said to be a natural one. This remark applies only to the method of *enumerating* by tens: the way now in use of *representing* numbers by nine digits and zero with the device of place, is a very refined invention.

the sign $+$ is used; it is read *plus* or *more*, and is placed between the numbers to be added.

Instead of the word *equals* the sign $=$ is used. This is called *the sign of equality.*

Thus, $5 + 3 + 1 = 9$ is read 5 *plus* 3 *plus* 1 *equals* 9, and expresses that the sum of 5, 3, and 1 is 9.

18. To add together two numbers, as 7 and 4, it is sufficient to add to the first, one after the other, all the ones which compose the second, in this way, 7 and 1 are 8, 8 and 1 are 9, 9 and 1 are 10, 10 and 1 are 11. This is what beginners do when they reckon by means of their fingers, or by strokes, saying 8, 9, &c., and stopping as soon as they have reckoned off 4. This method would always be troublesome, and impracticable if the numbers were very large. We begin, then, by learning to add promptly the first nine numbers to any given number. It will soon be seen that to one who can do this, addition never presents any difficulty; the learner, therefore, must proceed no further until he can answer at once such questions as, What is the sum of 9 and 7? 14 and 5? 27 and 8? 59 and 2?

19. In order to find the sum of 7 and 5, it makes no matter whether we add 5 to 7, or 7 to 5, the result in each case being 12; and the reason of this is plain. Make 7 strokes, and after them place 5 strokes; there is clearly the same number of strokes, whether we begin with 7 and add 5 to them, or begin with 5 and add 7 to them. In the same way, $7 + 6 + 5 = 5 + 6 + 7 =$ &c., the sum in each case being 18; and so for any other numbers. Hence:—

1 1 1 1 1 1 1   1 1 1 1 1

If we have to find the sum of two or more numbers, it is indifferent in what order we add them.

20. If we add 3 to 7, and then add 2 to the result, upon the whole we add 5 to 7; thus, $7 + 3 = 10$, $10 + 2 = 12$, and $7 + 5 = 12$. In the same way, since $16 = 10 + 6$, in order

1 1 1 1 1 1 1   1 1 1   1 1

to add 16 we may first add 10, and to the result add 6; to add 87 we may first add 7 and then 80, &c. Hence:—

Instead of adding any number at once, we may, if it is convenient, separate it into parts, and add these parts successively.

21. Three boys and two boys are five boys, 3 pens and 2 pens are five pens, 3 ones and 2 ones are five ones, 3 tens and 2 tens are 5 tens, &c.; but 3 boys and 2 girls are neither 5 boys nor 5 girls, 3 tens and 2 ones are neither 5 tens nor 5 ones, &c. Hence: —

We can only add together numbers of the same kind.

22. Let it be required to find the sum of 43 and 56. Now, since $56 = 50 + 6$, in place of adding 56 at once, we may first add 50, and then 6 (Art. 20.). Hence:—

$$
\begin{aligned}
43 + 56 &= 4 \text{ tens} + 3 \text{ ones} + 5 \text{ tens} + 6 \text{ ones} \\
&= 4 \text{ tens} + 5 \text{ tens} + 3 \text{ ones} + 6 \text{ ones} && \text{(Art. 19.)} \\
&= 9 \text{ tens} + 9 \text{ ones} \\
&= 99 && \text{(Art. 21.)}
\end{aligned}
$$

Again,

$$
\begin{aligned}
26 + 79 &= 2 \text{ tens} + 6 \text{ ones} + 7 \text{ tens} + 9 \text{ ones} && \text{(Art. 20.)} \\
&= 2 \text{ tens} + 7 \text{ tens} + 6 \text{ ones} + 9 \text{ ones} && \text{(Art. 19.)} \\
&= 9 \text{ tens} + 15 \text{ ones} && \text{(Art. 21.)} \\
&= 9 \text{ tens} + 10 \text{ ones} + 5 \text{ ones} && \text{(Art. 20.)} \\
&= 9 \text{ tens} + 1 \text{ ten} + 5 \text{ ones} \\
&= 10 \text{ tens} + 5 \text{ ones} = 1 \text{ hund.} + 5 \text{ ones} = 105.
\end{aligned}
$$

In the same way,

$$
\begin{aligned}
&2364 + 796 + 13 + 7802 \\
&= 2 \text{ thous.} + 3 \text{ hund.} + 6 \text{ tens} + 4 \text{ ones} \\
&\qquad\qquad + 7 \text{ hund.} + 9 \text{ tens} + 6 \text{ ones} \\
&\qquad\qquad\qquad\qquad + 1 \text{ ten} + 3 \text{ ones} \\
&+ 7 \text{ thous.} + 8 \text{ hund.} + 0 \text{ ten} + 2 \text{ ones} \\
&= 9 \text{ thous.} + 18 \text{ hund.} + 16 \text{ tens} + 15 \text{ ones} && \text{(Art. 19.)}
\end{aligned}
$$

= 9 thous. + 10 hund. + 8 hund. + 10 tens + 6 tens
+ 10 ones + 5 ones
= 9 thous. + 1 thous. + 8 hund. + 1 hund. + 6 tens
+ 1 ten + 5 ones
= 10 thous. + 9 hund. + 7 tens + 5 ones
= 10975.

23. In each of these examples it will be seen that we add all the ones together, all the tens together, &c., taking the tens of ones in the sum along with the tens, the tens of tens along with the hundreds, the tens of hundreds along with the thousands, &c. Hence the following

RULE.—*Write the numbers to be added under one another so that ones shall be under ones, tens under tens, hundreds under hundreds, &c. Then add up the first column to the right, and if the sum does not exceed 9, place the figure which expresses it under that column, but if the sum exceed 9, only write the right hand or ones' figure, and keep in memory the other figure, or figures, denoting the number of tens. Next, add together all the figures in the column of tens, taking in the number carried from the first row; put down the right hand figure, and carry the rest as before. Proceed in this manner to the last column, the whole of whose sum must be put down.*

Thus, to add by this rule, the numbers 2364, 796, 13, 7812, and 1904, we write these as in the margin, and begin by adding the ones' line: 4 and 2 are 6, and 3 are 9, and 6 are 15, and 4 are 19, 19 ones are 1 ten and 9 ones; place 9 under the ones, and carry 1 ten to the tens; 1 and 1 are 2, and 1 are 3, and 9 are 12, and 6 are 18, 18 tens are 1 hund. and 8 tens; place the 8 tens under the tens, and carry the 1 hund. to the hundreds; 1 and 9 are 10, and 8 are 18, and 7 are 25, and 3 are 28, 28 hundreds are 2 thousands and 8 hundreds; place 8 hundreds under the hundreds, and carry 2 thousands to the

$$\begin{array}{r} 2364 \\ 796 \\ 13 \\ 7812 \\ 1904 \\ \hline 12889 \end{array}$$

thousands; 2 and 1 are 3, and 7 are 10, and 2 are 12, 12 thous. = 2 thous. and 1 ten thous.; place 2 under the thousands, and 1 to the left of it, in the ten-thousands' place: thus, the sum is 12889.

In adding, the habit must be acquired of passing at once to the successive sums: thus, in working the above example, nothing more should be said, or thought, while ascending the first column, than 6, 9, 15, 19; put down 9 at once, and pass to the next column, saying only 2, 3, 12, 18; and so for the rest.

24. PROOF. — After having gone through any calculation, in order to assure ourselves that the result is correct, we may either simply review what has been done, or work the same example another way. This second operation is called the *proof.* The best way of verifying the result in addition is to add the columns of figures as before, but *downwards:* the two results should agree (Art. 19.). Since in the second operation the numbers are not added in the same order as before, we are not likely to commit the same error as in the former one, if an error has been made.

25. If the sum of the figures in each column were never more than 9, it would make no matter whether we commenced at the column of ones, or at any other column. But as it generally happens that many of these sums exceed 9, if we were to begin at the left we would frequently be obliged to retrace our steps in order to separate one of these into ones and tens, and to increase the sum obtained just before by the tens. But when we begin at the right, since 10 of any column are equal to 1 of that immediately to the left, this separation is easily performed as we pass from one column to the next.

SUBTRACTION.

26. There were 121 boys in the school, 17 have gone away, how many are there left? This number, whatever it may be, is called the *remainder;* in finding it we are said to *subtract*, that is, *take away*, 17 from 121, and the process is called SUBTRACTION.

Subtraction, then, *is an operation by which we take away the less of two numbers from the greater; the result of this operation is called the remainder.*

27. It is plain that the remainder expresses by how much the greater of the two numbers exceeds the less, that is, what must be added to the less to give the greater. Thus, the annexed arrangement of strokes shows that if 3 be taken from 8 the remainder is 5, or that 8 exceeds 3 by 5, or that 5 must be added to 3 to produce 8. For this reason the remainder is also called the *difference;* the difference of 8 and 3 is 5.

```
1 1 1 1 1 1 1 1
          1 1 1
1 1 1 1 1
```

28. To indicate subtraction, the sign — (*minus*) is placed between the two given numbers. Thus, if we wish to say that 3 taken from 8 leaves 5, we write $8 - 3 = 5$, which is read 8 *minus* 3 *equals* 5.

29. Since 6 and 3 are 9, $9 - 3 = 6$, $9 - 6 = 3$; since 7 and 9 are 16, $16 - 7 = 9$, $16 - 9 = 7$, &c. An ability to perform subtractions like these, which only require a knowledge of the sum of any two numbers, each not more than 9, will enable one to find the difference of any two numbers whatever. The pupil, therefore, must not proceed further until he can answer promptly such questions as, what is the difference of 13 and 7? what must be added to 3 to give 8? 17 minus 9?

30. If 2 be taken from 6, the remainder is 4; add 3 to each, then $9 - 5 = 4$; add 5 to each, then $11 - 7 = 4$;

and so on, the difference in every case being the same, 4. In the same way, $6 - 4 = 2$; add 10 to each, $16 - 14 = 2$; and so on, the difference always remaining the same, because what we add to one is compensated by an equal addition to the other. Hence: —

If two numbers be equally increased, their difference remains the same.

31. If we take 4 away and then 2, upon the whole we take 6 away; thus $9 - 4 = 5$, $5 - 2 = 3$, and $9 - 6 = 3$. In the same way, since $16 = 10 + 6$, to subtract 16 we may first subtract 10 and then 6; to subtract 254 we may first take away 4, then 50, then 200; and so on. That is,

Instead of taking away the whole of any number at once, we may, if we please, separate it into any convenient parts, and successively take away the several parts.

32. Since $7 + 5 = 12$, and $12 - 3 = 9$, therefore $7 + 5 - 3 = 9$; also $7 - 3 + 5 = 4 + 5 = 9$; and $5 - 3 + 7 = 2 + 7 = 9$: in each case the final result is 9. In the same way, $7 + 6 - 2 - 5 = 7 - 2 + 6 - 5 =$ &c., the result being always 6. That is,

If we have any number of successive additions and subtractions, it is indifferent how we change the order of these operations, provided they be all performed.

33. Six boys minus two boys = four boys, 6 ones − 2 ones = 4 ones, 6 tens − 2 tens = 4 tens, but the difference between 6 boys and 2 girls is neither 4 boys nor 4 girls, between 6 tens and 2 ones is neither 4 tens nor 4 ones. Hence: —

Subtraction can only be performed between numbers of like kind.

34. Let it be required to take 342 from 574. We have seen (Art. 31.), that instead of taking away 342 at once, we may take away first 300, then 40, then 2; hence we have,

$$574 - 342 = 5 \text{ hund.} + 7 \text{ tens} + 4 \text{ ones} - 3 \text{ hund.} - 4 \text{ tens} - 2 \text{ ones}$$

$= 5$ hund. $-$ 3 hund. $+$ 7 tens $-$ 4 tens
$+$ 4 ones $-$ 2 ones (Art. 32.)
$=$ 2 hund. $+$ 3 tens $+$ 2 ones (Art. 33.)
$= 232$.

We have here taken the ones, the tens, and the hundreds of the less number, respectively from the ones, the tens, and the hundreds of the greater, the sum of the remainders thus found forming the difference between the two given numbers.

Next, let us take 685 from 942. As before, instead of subtracting 685 at once, we take away successively 600, 80, and 5. Hence,

$942 - 685 = 9$ hund. $+$ 4 tens $+$ 2 ones $-$ 6 hund. $-$ 8 tens $-$ 5 ones $=$ 9 hund. $-$ 6 hund. $+$ 4 tens $-$ 8 tens $+$ 2 ones $-$ 5 ones, so that we may write the numbers in this way,

9 hund. 4 tens 2 ones
6 hund. 8 tens 5 ones,

and find the separate remainders as before. But we here encounter a difficulty which did not present itself in the first example. Although 942 is greater than 685, so that we can take the latter from the former, yet the ones and tens in 685 are respectively greater than those in 942, so that the separate subtractions on these cannot be performed. To obviate this, add 10 ones to the upper number, increasing the 2 ones to 12 ones, and to compensate for this (Art. 30.) add 1 ten, which is the same as 10 ones to the lower number, changing the 8 tens into 9 tens; also, add 10 tens to the upper number, so that the 4 tens become 14 tens, and to compensate for this, 1 hundred, which is the same as 10 tens, to the lower number, increasing the 6 hundreds to 7 hundreds. We have now for the first number, 9 hund. 14 tens 12 ones, and for the second, 7 hund. 9 tens 5 ones.

Now these two numbers have been formed by equally increasing the two given numbers; viz., the larger by 10 ones and 10 tens, or 110, and the smaller by 1 ten and 1 hundred, or 110; so that if we find the difference of the second pair it will be the same as the difference of the first pair (Art. 30.). Performing the subtractions, as in the former example, we obtain 2 hund. 5 tens 7 ones, or 257.

35. The learner will now see the reason of the following:

RULE. — *To subtract one number from another, place the smaller of the two below the larger, in such a way that ones shall be under ones, tens under tens, &c.; then, beginning at the right hand, take each figure in the lower line from the figure above it, placing the remainder underneath; but, when a figure in the lower line is greater than the one above it, increase the upper figure by ten, and then subtract the lower one, remembering, after that, to increase (mentally) the next lower figure by one. The number made up of the separate remainders thus found, is the difference of the two given numbers.*

Example. — Subtract 6075 from 31005. Arrange the numbers as the rule directs, and proceed as follows: 5 from 5, 0; 7 from 10, 3; 1 from 10, 9; 7 from 11, 4; 1 from 3, 2: the remainder is, therefore, 24930. We have really not taken 6075 from 31005, but 6075 increased by 1 hundred, 1 thousand, and 1 ten-thousand, from 31005 increased by 10 tens, 10 hundred, and 10 thousand, but as the additions to the one number are equal to the additions to the other, the remainder is just the same as if they were not made.

$$\begin{array}{r} 31005 \\ 6075 \\ \hline 24930 \end{array}$$

36. PROOF. — Add the remainder to the less number; the sum should be the greater (Art. 27.). Or, subtract the remainder from the greater number; the less number should be left (Art. 29.).

37. The addition, when necessary, of 10 to the upper figure will always make it greater than the lower one, since that cannot be more than 9, and instead of increasing it by

10 we may add 1 to the figure on its left; so that we are at once enabled to perform the partial subtraction, and to effect the requisite compensation. But if any other number, as 6, were added to the upper figure, we should have to add 6 to the corresponding lower one, so that the partial subtraction would still be impossible. With any other system of naming and representing numbers, we should, in like manner, add to the upper figure the *base* of the system (Art. 15.), and *one* to the lower figure next to the left.

38. If each figure in the lower number were less than the figure above it, it would be immaterial where we commenced the operation; but frequently this is not the case; so that, if we commenced at the left, we would, after adding 10 to any figure in the upper line, have to retrace our steps in order to increase the next preceding figure in the lower line by 1, and to make afresh at least one subtraction.

It is, therefore, best always to begin at the ones' column, so that such a step shall never be necessary.

MULTIPLICATION.

39. Before beginning this section the pupil must know the multiplication table thoroughly, so thoroughly as to be able at once to say when asked, what is the product of any two numbers, each not exceeding nine. In going through the preliminary exercises recommended in Art. 2., he would learn a considerable portion, if not the whole, of it before his introduction to slate-arithmetic. It is important to remember that the children should not be set to learn *a ready-made table*, but should be led, by proper questions, *to frame a table for themselves*. The teacher may proceed as follows: 2 and 2? how often is 2 taken to make 4? 2 times 2? 2 and 2 and 2? how often is 2 taken to make 6? 3 times 2? and so on.

40. There are 4 classes in the school, each of which contains 49 boys, how many boys are there altogether? That is, how many boys are 4 times 49 boys? This number, whatever it may be, is called the *product* of 49 and 5, in finding it we are said to *multiply* 49 by 5, and the process is called MULTIPLICATION. The number repeated, namely 49, is called the *multiplicand*, and 5, which shows how often it has to be repeated, the *multiplier*.

The multiplication of whole numbers, therefore, *is an operation which has for its end to repeat a certain number, called the multiplicand, as many times as there are ones in another number, called the multiplier. The result of this operation is called the product.*

41. Since the multiplier indicates the number of times the multiplicand has to be repeated, it must always be an abstract number; that is, it must be such a number as 3, or 5, or 14, &c., not such a number as £3, or 5 feet, or 14 oz., &c. Multiply 5 feet by 3, means repeat 5 feet 3 times, which is intelligible; but the phrase, multiply 5 feet by 3 feet is unmeaning, for what can be meant by repeating anything 5 feet times? The multiplicand may be abstract, in which case the product will be abstract; or concrete, in which case the product will be concrete, and of the same kind. Thus, 5 times 8 men $=$ 40 men, 5 times £8 $=$ £40, &c.; 5 times $8 = 40$, by which is meant that 8 things repeated 5 times give 40 things of the same kind, whatever they may be.

42. To indicate multiplication the sign $\times$ (*multiplied by*) is used. Thus, to show that 8 multiplied by 3 equals 24, we write $8 \times 3 = 24$.

43. When numbers are multiplied together they are called *factors* of the result. Thus, since $3 \times 7 = 21$, 3 and 7 are the factors of 21; since $3 \times 2 \times 5$, which means 3 multiplied by 2 and the product by 5, equals 30, 3, 2 and 5 are the factors of 30; and so on. And when we find what numbers must be multiplied together to produce a given number, we

are said *to resolve it*, or *to split it up*, into factors; thus, 21 and 30 have been resolved into factors in the above examples.

44. Multiply 2468 by 3, that is, find the sum of 2468, 2468, and 2468. Write down 2468 three times, and add: 8 and 8 are 16, and $8 = 24$; 2 and $6 = 8$, and $6 = 14$, and $6 = 20$; 2 and $4 = 6$, and $4 = 10$, and $4 = 14$; 2 and $1 = 3$, and $2 = 5$, and $2 = 7$. So that $2468 + 2468 + 2468$, or 3 times 2468, $= 7404$. Now the process of addition, by which we have arrived at this result, may be dispensed with, since we know, from the multiplication table, that $8 + 8 + 8$, or 3 times 8, $= 24$; that $6 + 6 + 6$, or 3 times 6, $= 18$; and that $2 + 2 + 2$, or 3 times 2, $= 6$. In finding the sum, therefore, we may say at once: 3 times $8 = 24$; 3 times $6 = 18$, and $2 = 20$; 3 times $4 = 12$, and $2 = 14$; 3 times $2 = 6$, and $1 = 7$. In working the example this way it is not necessary to write 2468 three times; it is sufficient to write it once, with the figure 3 under it to show how often it has to be repeated, as in the margin.

$$\begin{array}{r} 2468 \\ 2468 \\ \underline{2468} \\ 7404 \end{array} \qquad \begin{array}{r} 2468 \\ \underline{3} \\ 7404 \end{array}$$

Next, let it be required to multiply 50759 by 8.

$$\begin{array}{r} 50759 \\ \underline{8} \\ 406072 \end{array}$$

We proceed as follows: 8 times $9 = 72$; 72 ones $=$ 7 tens and 2 ones; place 2 in the place of ones and carry 7 tens in memory: 8 times 5 tens $=$ 40 tens, and 7 tens $=$ 47 tens; 47 tens $=$ 4 hund. and 7 tens; write 7 in the place of tens, and carry 4 hundred: 8 times 7 hundred $=$ 56 hundred, and 4 hundred $=$ 60 hundred $=$ 6 thousand; place 0 in the place of hundreds, to keep the other figures in their proper places, and carry 6 thousand: there are no thousands to multiply, but 6 thousands were carried from the preceding product, we therefore write 6 in the place of thousands: lastly, 8 times 5 ten-thousands $=$ 40 ten-thousands $=$ 4 hundred thousand; we therefore place 0 in the place of ten-thousands, and 4 in the hundred-thousands' place, that is, we write 40 to the

left of the preceding figures. Thus, the required product is 406072. In working, we need make no mention of tens, hundreds, &c., but simply say, 8 times 9 = 72; 8 times 5 = 40, and 7 = 47; 8 times 7 = 56, and 4 = 60; 8 times 0 = 0, and 6 = 6; 8 times 5 = 40. An expert Calculator does not say, or think, all this, but arrives instantly at the results 72, 47, 60, 6, 40.

Multiplication, then, is a short way of doing Addition, when the numbers to be added are all the same. When, in multiplying 50759 by 8, we say 8 times 9 = 72, 8 times 5 = 40, &c., we are merely saying that $9 + 9 + 9 + 9 + 9 + 9 + 9 + 9 = 72$, that 8 fives added together amount to 40, &c. In making, before learning, the multiplication table, we performed the additions, the results of which we are using now. Did we not remember these, the additions would now have to be performed, and an example in multiplication would be in appearance, what it is in reality, an example in addition. The process of making and learning the multiplication table is a process in which we undertake a certain amount of present labour to avoid much greater labour afterwards. It will soon be seen that two very large numbers can easily be multiplied together; the labour of solving the same example by addition would be intolerable.

The learner will now see the reason of the following

Rule.—*To multiply a number of several figures by a number of one figure, multiply successively, passing from right to left, each figure of the multiplicand by the multiplier, putting down the ones in order, and carrying the tens, as in Addition.*

45. Place 4 ones in a horizontal line, and repeat the line 3 times. Altogether, therefore, we have 3 times 4 ones, or the product of 4 by 3. Now if the rows are taken vertically, instead of horizontally, the same assemblage of ones is made up of 3 ones repeated 4 times, or is the product of 3 by 4. So that

1 1 1 1
1 1 1 1
1 1 1 1

$4 \times 3 = 3 \times 4$. In the same way we may show that $6 \times 7 = 7 \times 6$; and similarly for any other two numbers. Hence:

The product of two numbers is not changed by changing the order of the factors.

46. Again, $5 \times 3 = 15$, $15 \times 2 = 30$; $5 \times 2 = 10$, $10 \times 3 = 30$; and $5 \times 6 = 30$. So that either $5 \times 3 \times 2$ or $5 \times 2 \times 3 = 5 \times 6$. The annexed arrangements of ones will illustrate this. It will be seen that in the first, 5 ones are taken 3 times and the resulting assemblage of ones, that is 5×3, 2 times; that in the second, 5 ones are taken 2 times, and the result, that is 5×2, repeated 3 times; and that in either case, altogether, 5 is taken 6 times. This follows from the fact that $3 \times 2 = 2 \times 3$. In the same way, $5 \times 24 = 5 \times 3 \times 8$, or $5 \times 8 \times 3$, or $5 \times 3 \times 4 \times 2$, or $5 \times 4 \times 3 \times 2$, &c.; and similarly in other cases. Hence:—

1 1 1 1 1
1 1 1 1 1
1 1 1 1 1

1 1 1 1 1
1 1 1 1 1
1 1 1 1 1

1 1 1 1 1
1 1 1 1 1

1 1 1 1 1
1 1 1 1 1

1 1 1 1 1
1 1 1 1 1

Instead of multiplying at once by any number, we may, if we can, split it up into factors, and multiply successively by these in any order.

47. The figure 8 represents 8 ones, 80 represents 8 tens, or 8 times 10, which equals 10 times 8 (Art. 45.); therefore $80 = 8 \times 10$. Again, 800 represents 8 hundred, or 8 times 100, which equals 100 times 8; therefore $800 = 8 \times 100$. In the same way it may be shown that $8000 = 8 \times 1000$, &c. From these and similar examples it follows that

To multiply any whole number by a number expressed by 1 *with ciphers following, it is sufficient to write as many ciphers after the multiplicand as there are after the* 1.

Thus:— $215 \times 10 = 2150$,
$215 \times 100 = 21500$,
$215 \times 1000 = 215000$,
$215 \times 10000 = 2150000$, &c.

48. Since $30 = 3 \times 10$, therefore $87 \times 30 = 87 \times 3 \times 10$ (Art. 46.) $= 261 \times 10 = 2610$ (Art. 47.); $87 \times 300 = 87 \times 3 \times 100 = 261 \times 100 = 26100$; $87 \times 3000 = 87 \times 3 \times 1000 = 261 \times 1000 = 261000$, &c. When a number is terminated by ciphers, that part of it which precedes the ciphers is called the *significant* part: thus in 30, 300, 3000, &c., respectively, 3 is the significant part; 17 is the significant part in 17000, and 205 in 2050. Hence:—

When the multiplier has ciphers at the end of it, multiply by the significant part, and to the product affix the ciphers; or, write down the ciphers, and then, passing to the left, multiply by the significant part.

Ex.—Multiply 298463 by 9000. The work will stand as follows:—

$$\begin{array}{r} 298463 \\ 9000 \\ \hline 2686167000 \end{array}$$

49. Just as 4 boys and 2 boys are 6 boys, 4 pens and 2 are 6 pens, &c., so 4 fives and 2 fives are 6 fives, that is, $5 \times 4 + 5 \times 2 = 5 \times 6$. The same appears from actual multiplication: $5 \times 4 = 20$, $5 \times 2 = 10$, $20 + 10 = 30$, and $5 \times 6 = 30$. Again, 200 boys and 30 boys and 7 boys are 237 boys; so, 200 eighteens and 30 eighteens and 7 eighteens are 237 eighteens, that is, $18 \times 200 + 18 \times 30 + 18 \times 7 = 18 \times 237$. And similarly in other cases. Hence:—

Instead of multiplying at once by any number, we may separate it into any convenient parts, multiply the multiplicand separately by each part, and take the sum of the several products.

The pupil must not confound the two distinct principles implied severally in the equations, $5 \times 6 = 5 \times 3 \times 2$, $5 \times 6 = 5 \times 4 + 5 \times 2$.

50. Let it now be required to multiply 87049 by 395. Since $395 = 5 + 90 + 300$, therefore, (Art. 49.) $87049 \times$

$395 = 87049 \times 5 + 87049 \times 90 + 87049 \times 300 = 435245 + 7834410 + 26114700$, (Art. 48.) $= 34384355$. The work may stand thus :—

$$\begin{array}{r}
87049 \\
\underline{395} \\
435245 = 5 \text{ times } 87049 \\
7834410 = 90 \;\;,, \;\; 87049 \\
\underline{26114700} = \underline{300} \;\;,, \;\; 87049 \\
34384355 = 395 \;\;,, \;\; 87049
\end{array}$$

It is clear that the ciphers at the ends of the lines may be omitted provided the positions of the other figures be not altered. This is generally done in practice ; so that the work stands thus :—

$$\begin{array}{r}
87049 \\
\underline{395} \\
435245 \\
783441 \\
\underline{261147} \\
34384355
\end{array}$$

It will be observed that the first figure of the second line is in the same place as the multiplier, 9, which produces that line, viz. the tens' place; and that the first figure of the third line, viz. 7, is in the same place, the hundreds' place, as the multiplier, 3, which produces that line.

Next, multiply 87049 by 300905. We proceed as before.

$$\begin{array}{r}
87049 \\
\underline{300905} \\
435245 = 87049 \times 5 \\
78344100 = 87049 \times 900 \\
\underline{26114700000} = 87049 \times \underline{300000} \\
26193479345 = 87049 \times 300905
\end{array}$$

The ordinary form of working, which is substantially the same as the preceding, only differing from it in the omission of ciphers at the ends of the lines, is as follows :—

$$\begin{array}{r}
87049 \\
\underline{300905} \\
435245 \\
783441 \\
\underline{261147} \\
26193479345
\end{array}$$

It will be seen here also, that the first figure of each line is in the same column as the multiplier which produces that line. Thus, the first multiplier, 5, is in the first column, that is, in the ones' place, so is the first figure of the first line; the second multiplier, 9, is in the third column, that is, in the hundreds' place, so is the first figure, 1, of the second line; the third multiplier, 3, and the first figure of the third line, 7, are both in the sixth place. When many cyphers occur in the multiplier, it is, perhaps, the best way to multiply by the first, that is, the complete method. The learner will now see the reason of the following

RULE. — *When the multiplier consists of more than one figure, multiply the multiplicand successively by each significant figure of the multiplier, taking care to place the partial products under one another in such a way that the first figure of each, passing from right to left, shall be in the same line as that figure of the multiplier which has produced it : the total product is the sum of the partial products.*

51. Multiply 203400 by 37000. Since $203400 = 2034 \times 100$, and $37000 = 37 \times 1000$, therefore, $203400 \times 37000 = 2034 \times 100 \times 37 \times 1000 = 2034 \times 37 \times 100 \times 1000 = 2034 \times 37 \times 100000$; that is, we have simply to multiply 2034 by 37, and to the result affix five ciphers. The work stands thus:

```
 203400
  37000
 14238
 6102
 7525800000.
```

Hence the following

Rule. — *When the multiplicand and multiplier are terminated by ciphers, multiply together the significant parts, and affix to the result as many ciphers as there are at the end of the multiplicand and of the multiplier.*

52. Proof. — Multiply together the same two numbers, reversing the order of the factors, that is, taking the multiplicand for multiplier, and the multiplier for multiplicand: the second result should agree with the first (Art. 45.)

53. In finding the partial products, it is necessary, for a reason precisely similar to that given in Art 25., for the order observed in Addition, to take the figures of the multiplicand from right to left, but the figures of the multiplier may be taken in any order to multiply by. In the following example they are taken from left to right, just reversing the usual order.

```
   346
   297
 69200 = 200 times 346
 31140 =  90   ,,   ,,
  2422 =   7   ,,   ,,
102762 = 297   ,,   ,,
```

Omitting the ciphers we have:

```
   346
   297
 692
 3114
  2422
102762
```

It will be observed that this form is in accordance with the general rule given in Art. 50., inasmuch as the first figure, passing from right to left, of each partial product is in the same column as the figure of the multiplier which produces that product. It is clearly most convenient and natural to take the figures of the multiplier either directly from right to left or from left to right, and since the usual course agrees with the order in which the figures of the multiplicand are taken, it is to be preferred.

DIVISION.

54. I wish to divide 54 marbles equally amongst 9 boys, how many must I give to each?

In finding this number we are said to *divide* 54 marbles by 9, that is, to separate them into 9 equal parts; the number divided is called the *dividend;* the number by which it is divided, that is, the number of equal parts into which it is separated, the *divisor;* the result of the operation, that is, the value of each part, the *quotient;* and the process itself Division.

What we have to find in the above example is that number of marbles which repeated 9 times will give 54 marbles. From the multiplication table we know that this number is 6; the required quotient, therefore, is 6 marbles.

Again, to how many boys can marbles be given, if there are 54 altogether, and each boy receives 9?

The number may be found as follows; first, take 9 marbles from 54, that is, give *one* boy his share; take 9 from the remainder, that is, give a *second* boy his share, and so on. It will be seen that after 6 boys have received their

shares, there are no marbles left; so that the required number of boys is 6. Hence 9 marbles can be taken 6 times out of 54 marbles, or 54 marbles *contain* 9 marbles 6 times. This result may also be found from the multiplication table, for all we have to ask ourselves is, how often must 9 marbles be taken to make 54 marbles? The answer is 6 times.

54	
9	1
45	
9	2
36	
9	3
27	
9	4
18	
9	5
9	
9	6
0	

The second process, as well as the first, is called Division; but it will be seen that they differ in meaning, although the way in which they obtain their respective results, viz. by reference to the multiplication table, is the same. In the one case, we divide 54 marbles into 9 equal parts; in the other, we find how often 54 marbles contain 9 marbles; that is, in the first, having the number of equal parts given, we find the value of each part, in the second, having given the value of each part, we find the number of parts. In both cases the result is 6; but in the first it is 6 marbles, a *concrete* number, in the other, an *abstract* number, 6, that is 6 times. In both, also, the operation is the reverse of Multiplication. And this is the true meaning of Division. It is a process by which, having the product of two numbers given, and one of these, we find the other. Divide 48 by 6. To answer this, we have simply to ask ourselves what 6 must be multiplied by to produce 48. Answer, 8. Hence, 48 divided by $6 = 8$. If it is £48 to be divided by 6, the quotient is £8; if it be £48 to be divided by £6, the quotient is 8.

Division, then, is *an operation which has for its end, having given the product of two numbers, and one of these numbers, to find the other. The given product is called the dividend, the given factor the divisor, and the required factor the quotient.*

55. It is easily seen that the dividend and divisor may be both concrete numbers, in which case the quotient is an

abstract number; or, the dividend may be concrete and the divisor abstract, in which case the quotient is concrete and of the same kind as the dividend; or, both dividend and divisor may be abstract, in which case the quotient is also abstract; but it is impossible for the dividend to be abstract, and the divisor concrete. Such an expression as 48 divided by £6 is unmeaning.

56. To indicate division the sign $\div$ (*divided by*) is used. Thus to show that 48 divided by 6 equals 8, we write $48 \div 6 = 8$.

57. Before the pupil proceeds any farther he must be able promptly to answer such questions as the following: 6×8? How many eights are there in 48? What, then, is $48 \div 8$? What is 48 called? 6? 8? The results should occasionally be written on the board, thus: $6 \times 8 = 48$, $48 \div 6 = 8$, $48 \div 8 = 6$.

58. It has been seen that the object of division is, having given the product of two numbers (the dividend), and one of these numbers (the divisor), to find the other (the quotient). It follows from this that

Divisor $\times$ *Quotient, or, Quotient* $\times$ *Divisor* $=$ *Dividend.*

But let it be required to divide 52 by 6, that is, to find what 6 must be multiplied by to produce 52. Now, from the multiplication-table we know that $6 \times 8 = 48$, $6 \times 9 = 54$; so that no whole number multiplied into 6 will give 52; 48 is the nearest number *below* 52 that can be produced by multiplication of 6, and to get 52 we must add 4 to this. Hence 52 contains 8 sixes and 4 ones more. Here 4 is called the *remainder;* so that in division, *the remainder expresses how much more the dividend is than the product of the divisor and quotient.* In the same way, since $27 = 7 \times 3 + 6$, 27 contains 3 sevens and 6 ones more or 6 ones over, that is, $27 \div 7 = 3$ and 6 over; since $83 = 9 \times 9 + 2$, therefore $83 \div 9 = 9$ and 2 over, &c. Hence, when in division there is a remainder,

Dividend $=$ *Divisor* $\times$ *Quotient* $+$ *Remainder.*

It is clear that the remainder is always less than the divisor.

59. The number of farthings in 4 pence + the number in 6 pence = the number in 10 pence, or the number of feet in 4 yards + the number in 6 yards = the number in 10 yards; in just the same way, the number of twos in 4 + the number in 6 = the number in 10, or, $4 \div 2 + 6 \div 2 = 10 \div 2$; and we see by actual division that this is the case, for $4 \div 2 = 2$, $6 \div 2 = 3$, $3 + 2 = 5$, and $10 \div 2 = 5$. In the same way,

since	6 contains	2 threes,		
	9	„	3	„ ,
	12	„	4	„ ,
therefore,	27	„	9	„ ;

or, since $27 = 6 + 9 + 12$, therefore $27 \div 3 = 6 \div 3 + 9 \div 3 + 12 \div 3$. From examples like these it follows, that

If we have to divide one number by another, we may separate the dividend into any convenient parts, divide each separately by the divisor, and take the sum of the several quotients.

60. As £$6 \div 3 =$ £2, 6 marbles $\div 3 = 2$ marbles, &c., so 6 tens $\div 3 = 2$ tens, 6 hundreds $\div 3 = 2$ hundreds, 6 thousands $\div 3 = 2$ thousands, &c.; that is, $60 \div 3 = 20$, $600 \div 3 = 200$, $6000 \div 3 = 2000$, &c. In the same way, 18 tens $\div 6 = 3$ tens, or $180 \div 6 = 30$, $1800 \div 6 = 300$, $18000 \div 6 = 3000$, &c.; and similarly in other cases of like kind.

61. Divide 6936 by 3. We cannot solve this, and similar questions, at once, in the way, for example, that we divide 27 by 3, because we can only retain in our memories the products of 3 by a very limited set of numbers; but observing that $6936 = 6000 + 900 + 30 + 6$, we can readily find the required quotient by dividing each of these parts into which we have separated 6936 by 3 (Art. 60.), and taking the sum of the several quotients (Art. 59.). Now $6000 \div 3 = 2000$, $900 \div 3 = 300$, $30 \div 3 = 10$, $6 \div 3 = 2$. And since

	6000 contains 3	2000 times,	
	900 „	300 „ ,	
	30 „	10 „ ,	
and	6 „	2 „ ,	
therefore,	6936 „	2312 „ ,	

or $6936 \div 3 = 2312$. It is customary to place the divisor, dividend, and quotient, as in the margin, and to work as follows: 6 by $3 = 2$ (it is not necessary to say 6 thousand by $3 = 2$ thousand, because by putting the 2 under the 6, that is, in the thousands' place, we mark its value); 9 by $3 = 3$ (by placing this under the 9, we show that it is 300, therefore we need not say, in working, 9 hundred by $3 = 3$ hundred); 3 by $3 = 1$ (we do not say 3 tens by $3 = 1$ ten, because we put the 1 under the 3, that is, in the tens' place); $6 \div 3 = 2$, which falls in the ones' place.

$$\begin{array}{r} 3\overline{)6936} \\ \overline{2312} \end{array}$$

Next, let it be required to divide 4792 by 6. As before, we separate the dividend into parts, in order to divide each separately by 6: $4792 = 4000 + 700 + 90 + 2$. If we have to divide £4 into 6 equal parts, each part will be less than £1, will be so many shillings; so that before the division can be performed, £4 must be turned into shillings. In like manner, 4 thous. divided by 6 will give less than 1 thous., the result will be in hundreds; hence, for the same reason that £4 is expressed in shillings, 4 thous. is turned into hundreds: $4000 = 40$ hunds., and there are 7 hunds. more; hence we have $4792 = 4700 + 90 + 2$. Just as £47 $\div$ 6 = £7 and £5 over, so 47 hund. $\div$ 6 = 7 hund. and 5 hund. over, that is, the next number of hundreds below 47 divisible by 6 is 42, and, as our object is to separate the dividend into parts each of which is divisible by 6, we now write $4792 = 4200 + 500 + 90 + 2$. Since 5 hund. divided by 6 will not give so much as 1 hund., we call them 50 tens, which with the 9 tens following make 59 tens; 59 tens by 6 = 9 tens and 5 tens over, that is, the next number of tens below 59 divisible by 6 is 54; we have now $4792 = 4200 +$

540 + 50 + 2 = 4200 + 540 + 52. Lastly, 52 by 6 = 8 and 4 over, that is, the next number below 52 divisible by 6 is 48; we have therefore

$$4792 = 4200 + 540 + 48 + 4;$$

so that 4792 is not exactly divisible by 6, but exceeds the number next below it which is so divisible, by 4. Since

$$\begin{aligned} 4200 \div 6 &= 700, \\ 540 \div 6 &= 90, \\ 48 \div 6 &= 8, \\ \text{therefore, } \overline{4788} \div 6 &= \overline{798}; \end{aligned}$$

hence, 4792 divided by 6 gives a quotient of 798 and a remainder of 4, or 4792 ÷ 6 = 798 and 4 over.

$$\begin{array}{l} 6)\underline{4792} \\ \quad 798 \ . \ 4 \end{array}$$

If the above explanation be reviewed, it will be seen that the quotient is obtained in the following manner: 4700 ÷ 6 = 700 and 500 over; 500 = 50 tens, and 9 tens = 59 tens; 59 tens ÷ 6 = 9 tens and 5 tens over; 50 + 2 = 52; 52 ÷ 6 = 8 and 4 over. It may be explained, as in the preceding example, that, in working, no mention need be made of hundreds, &c., and that we may proceed in this way: 47 by 6 = 7 and 5 over, 59 by 6 = 9 and 5 over, 52 by 6 = 8 and 4 over.

62. In going through the two examples just solved, we have performed mentally all the operations required, and put down only the various quotients, which are all we want. If, with respect to the last example, every step be exhibited on paper, the work will appear as in the margin, where the quotient is put to the right of dividend, and the divisor as before. In the first process, having found that number, 7, by which 6 must be multiplied to give the product, 42 next below 47, we put the 7 in the quotient place, and carry the excess, 5, of 47 over that product *in memory;* in the second, having written the quotient, 7, found as before, in the place assigned it, we subtract *on paper*

$$\begin{array}{l} 6)4792(798 \\ \quad \underline{42} \\ \quad\ \ 59 \\ \quad\ \ \underline{54} \\ \quad\quad 52 \\ \quad\quad \underline{48} \\ \quad\quad\ \ 4 \text{ remr.} \end{array}$$

the product of 6 by it from 47; in the first, we next say 59 by 6 = 9 and 5 over, the number 59 being formed by placing the remainder *before* the next figure of the dividend, in our minds; in the second, we form the same number, 59, by placing the next figure, 9, of the dividend *after* the remainder (this is called *bringing down* the 9); the quotient is then found as in the first process, and the product of 6 by it subtracted on paper from 59; and so on until the work is done.

Examples worked in the first way are said to be done by SHORT DIVISION; in the second way, by LONG DIVISION. In both forms precisely the same operations are gone through, but in the second those are displayed to the eye, which in the first are performed mentally. When the divisor is small, as in dividing 53 by 7, we can tell the quotient and the remainder at once, from our knowledge of the multiplication table; but when the divisor is large we are obliged, if not much practised in Arithmetic, to try two or three numbers, mentally, before the proper quotient is found, and then to multiply and subtract on paper, in order to obtain the remainder. Thus, in the example 2934 ÷ 398, we find, on trial, that 7 times 398 falls short of 2934, and that 8 times 398 exceeds it; therefore, the quotient is 7; to find the remainder, 398 is multiplied by 7, and the product subtracted from 2934. It would not be easy to do all this mentally; hence, for small divisors the first form is used, and for large divisors the second. In general, Short Division is used with divisors not exceeding 12 (this number is determined by the ordinary limit of the multiplication table); and Long Division with divisors above 12; but expert arithmeticians can divide in the short way by numbers much larger than 12.

```
398)2934(7
    2786
    ----
     148
```

If, as happens sometimes when the divisor is large, too small a number be taken for quotient, the remainder will exceed the divisor (Art. 58.). Whenever, therefore, the

remainder exceeds the divisor, the number taken for quotient is less than the real quotient, and a larger number must be tried.

Divide 1685934 by 234. Working this example in the second way above described, it will be seen that when the proper figure, 3, is brought down to the second remainder, the resulting number, 113, is less than the divisor; in such a case we place a cipher in the quotient, bring down the next figure, and proceed as before. To explain this, we shall go over the example as in the last article. The first partial dividend, 1685, is really 1685 thousands; the second, 479 hundreds, the third, 113 tens; and the fourth, 1134 ones, or, simply 1134. The complete process is: 1685 ÷ 234 = 7 and 47 over, therefore, 1685 thous. ÷ 234 = 7 thous. and 47 thous. over; 47 thous. = 470 hund., which added to 9 hund., the next part of the whole dividend, = 479 hund.; 479 ÷ 234 = 2 and 11 over, therefore 479 hund. ÷ 234 = 2 hund. and 11 hund. over; 11 hund. = 110 tens, 110 tens + 3 tens = 113 tens; 113 is less than 234, hence, just as 113 shillings ÷ 234 give less than a shilling, 113 tens ÷ 234 give less than a ten, so that there are no tens in the quotient; 1130 + 4 = 1134, which divided by 234 gives 4 and 198 over.

```
234)1685934(7204
    1638
    ----
     479
     468
     ----
     1134
      936
      ---
      198 remr.

234)1685934
    -------
    7204..198
```

From this explanation and the annexed form, it is clear that we have resolved 1685934 into 1638000 + 46800 + 936 + 198, divided each part, except the last, by 234, and taken the sum of the several quotients. The

```
234)1685934
    1638000|   7000
    -------
      47934
      46800|    200
      -----
       1134
        936|      4
        ---    ----
        198    7204
```

first part contains 234, 7000 times; the second, 200 times; the third, 4 times; and 198 is left. So that 1685934 ÷ 234 = 7204 and 198 over; that is, 1685934 exceeds the product of 234 and 7204 by 198.

63. The following two rules will now be understood:

RULE i. SHORT DIVISION. —*Take, mentally, enough figures from the left of the dividend to compose a number not less than the divisor; find how often this number contains the divisor, write the figure obtained in the quotient place, and carry the remainder in memory. Put, mentally, the remainder before the figure of the complete dividend following the first partial dividend, divide the resulting number by the divisor, write the new quotient to the right of the first, and carry the remainder in memory. Proceed in this way, always writing the new quotient to the right of the preceding one, and putting, mentally, the remainder before the next figure of the complete dividend to form a new partial dividend, until every figure has been taken in. If any of the partial dividends be less than the divisor, put a cipher in the quotient and treat the dividend as a remainder.*

RULE ii. LONG DIVISION. — *Take, mentally, enough figures from the left of the dividend to compose a number not less than the divisor; this is the first partial dividend, and it must contain, at the least, as many figures as the divisor, or, at the most, one more. Find how often this number contains the divisor, write the figure obtained in the quotient place, multiply the divisor by this figure, and subtract the product from the first partial dividend. To the right of the remainder bring down the figure of the complete dividend next after the first partial dividend, and the number thus formed is the second partial dividend. Divide this by the divisor, write the new quotient to the right of the first, multiply the divisor by it, and subtract the product from the second partial dividend. To the remainder bring down the*

next figure of the complete dividend, not brought down, and the third partial dividend is formed, which must be divided for the third figure of the quotient. Proceed in this way, always bringing down to the remainder the next figure of the complete dividend, not brought down, to form a new partial dividend, and writing the quotient obtained from this to the right of the preceding quotient figures, until all the figures of the dividend have been successively brought down. If it happens that one of the partial dividends is less than the divisor, write a cipher in the quotient, bring down the next figure, and continue the operation as before.

64. PROOF.—Multiply the divisor by the quotient, or the quotient by the divisor; if there is no remainder, the product should be the dividend; if there is a remainder, add it to the product, and the sum should be the dividend (Art. 58.).

65. It will be noticed that the order of operation in division is from left to right, whereas in the three preceding rules it is from right to left. To see the reason of this, it is sufficient to consider attentively the explanations given in Arts. 61 and 62. From these it will readily be understood, that the dividend is the sum of the partial products of the divisor by the ones, tens, hundreds, &c. of the quotient, or the sum of these products and the remainder, when there is one; and that, in the ordinary process of division, the product of the divisor by the highest order of figure in the quotient is found in the first partial dividend, by the next highest, in the second partial dividend, &c.; so that from these dividends the figures of the quotient may at once be found in order. But if we were to begin at the right hand, the partial products of the divisor by the ones, tens, hundreds, &c. of the quotient would not appear, on account of the way in which they are combined when added to form the dividend; and the figures of the quotient, therefore, could not be directly found. In other words, since the remainder

from any partial division must be turned into ones' of the next *lower* order, and added to the ones of that order in the complete dividend, to form the partial dividend which when divided will give the figure of that order in the quotient, it is necessary in division to proceed from left to right, as in addition it is necessary to proceed in the contrary direction, because the tens of any partial sum must be turned into ones of the next *higher* order, to be added to the ones of that order in the total. In such examples as $6936 \div 3$, $20800462 \div 2$, &c., we need not necessarily begin at the left hand.

66. If 48 be divided by 6, the quotient is 8. Now $48 \div 3 = 16$, and $16 \div 2 = 8$; also, $48 \div 2 = 24$, $24 \div 3 = 8$. So that if 48 be divided by 3, and the quotient by 2, or first by 2, and the quotient by 3, the result is the same as if it be divided at once by 6, which is 2×3, or 3×2. Again, $120 \div 5 = 24$, $24 \div 4 = 6$, $6 \div 3 = 2$; or, $120 \div 3 = 40$, $40 \div 5 = 8$, $8 \div 4 = 2$, &c.; and $120 \div 60 = 2$. But $60 = 5 \times 4 \times 3$; hence the same result is obtained whether we divide at once by 60, or successively by its factors in any order. To illustrate this, take a line and divide it into 3 equal parts, then divide one of these, that is the quotient, into 2 equal parts; or, first divide the line into 2, and then each of these into 3 equal parts: it is clear that in the first case, upon the whole, the line has been divided into twice three, and in the second into three times two, that is, in both cases, into six equal parts. And similarly for any other *composite* number, that is, a number which can be resolved into two or more factors. Hence,

In place of dividing at once by a composite number, we may divide successively by its various factors.

This follows directly from what was proved in Art. 46, as will be seen in the examples solved below.

Ex.—Divide 68769 by 27. Since $27 = 3 \times 9$, we may divide successively by 3 and 9.

$$\begin{array}{r|l} 3 & 68769 \\ \hline 9 & 22923 \\ \hline & \quad 2547 \end{array}$$

$$\begin{array}{l} 27)68769(2547 \\ \quad \underline{54} \\ \quad 147 \\ \quad \underline{135} \\ \quad\ \ 126 \\ \quad\ \ \underline{108} \\ \quad\quad 189 \\ \quad\quad \underline{189} \\ \quad\quad \ldots \end{array}$$

Bearing in mind that dividend = quotient × divisor, we may show that 68769 has been, in the first process, divided by 27, as follows:

$$\begin{aligned} 22923 &= 2547 \times 9, \\ 68769 &= 22923 \times 3 \\ &= 2547 \times 9 \times 3 \\ &= 2547 \times 27, \end{aligned}$$

therefore, $68769 \div 27 = 2547$.

Divide 13894 by 36. Since $36 = 9 \times 4$, or 6×6, or 12×3, &c., we may divide in several ways; we shall divide first by 4, and then by 9 (it is better to take the smaller number first).

$$\begin{array}{r|l} 4 & 13894 \\ \hline 9 & 3473 \cdot\cdot 2 \\ \hline & \ \ 385 \cdot\cdot 8 \end{array} \Big\} 34$$

Here in dividing by 4 there is a remainder of 2, and in dividing by 9, a remainder of 8; the question is, what is left in dividing 13894 by 36? To answer this, we have simply to remember that dividend = divisor × quotient + remainder, and what was proved in Art. 46. Hence we have

$$\begin{aligned} 3473 &= 385 \times 9 + 8, \\ 13894 &= 3473 \times 4 + 2 \\ &= 385 \times 9 \times 4 + 8 \times 4 + 2 \,* \\ &= 385 \times 36 + 32 + 2 \\ &= 385 \times 36 + 34, \end{aligned}$$

* A property of numbers is here assumed, which has not been di-

that is, 13894 exceeds the product of 385 and 36 by 34, hence 13894 divided by 36 gives a quotient of 385 and a remainder of 34. Now $34 = 8 \times 4 + 2$: hence,

When there are two successive divisors, *the total remainder = 2nd remainder × 1st divisor + 1st remainder.*

Again, divide 24631 by 126. Since $126 = 3 \times 6 \times 7$, we may divide successively by these factors.

$$\begin{array}{r|l} 3 & 24631 \\ \hline \end{array}$$
$$\left.\begin{array}{r} 6\,|\,8210\cdot\cdot 1 \\ 7\,|\,1368\cdot\cdot 2 \\ 195\cdot\cdot 3 \end{array}\right\} 61$$

---

rectly proved. Let it be required to find 3 times the sum of 15, 7, 24, and 68. Bearing in mind the principles proved in Arts. 20. and 19., we have for the required result

$$\begin{aligned} & 15 + 7 + 24 + 68 \\ + & 15 + 7 + 24 + 68 \\ + & 15 + 7 + 24 + 68 \\ = & 15 + 15 + 15 \\ + & 7 + 7 + 7 \\ + & 24 + 24 + 24 \\ + & 68 + 68 + 68 \end{aligned}$$

= 3 times 15 + 3 times 7 + 3 times 24 + 3 times 68. So that in place of adding together 15, 7, 24, and 68, and multiplying the result by 3, we may multiply each of these numbers by 3, and add together the several products. To represent 3 times the sum of 15, 7, 24, and 68, we use the expression

$$(15 + 7 + 24 + 68) \times 3;$$

hence we have proved that

$$(15 + 7 + 24 + 68) \times 3 = 15 \times 3 + 7 \times 3 + 24 \times 3 + 68 \times 3.$$

In precisely the same way it may be shown that

$$(11 + 19 + 26) \times 13 = 11 \times 13 + 19 \times 13 + 26 \times 13;$$

and so on. Therefore,

The product of the sum of several numbers by another number is equal to the sum of the products obtained by multiplying each of the former by the latter.

The rule in Art. 44. is based upon this principle.

We proceed to find the remainder as before:

$$
\begin{aligned}
1368 &= 195 \times 7 + 3, \\
8210 &= 1368 \times 6 + 2 \\
&= 195 \times 7 \times 6 + 3 \times 6 + 2, \\
24631 &= 8210 \times 3 + 1 \\
&= 195 \times 7 \times 6 \times 3 + 3 \times 6 \times 3 + 2 \times 3 + 1 \\
&= 195 \times 126 + 54 + 6 + 1 \\
&= 195 \times 126 + 61;
\end{aligned}
$$

therefore, $24631 \div 126 = 195$ and 61 over. It will be seen that the remainder is obtained by multiplying the 3rd remainder successively by the 2nd and 1st divisors, the 2nd remainder by the 1st divisor, and adding together these two products and the 1st remainder. In this way we obtain the following rule for finding the final remainder when there is any number of successive divisors.

Multiply the 2nd remainder by the 1st divisor, the 3rd remainder successively by the 2nd and 1st divisors, the 4th remainder successively by the 3rd, 2nd, and 1st divisors, &c.; then add together all these products and the 1st remainder.

67. Since $173 = 170 + 3 = 10 \times 17 + 3$, therefore, $173 \div 10 = 17$ and 3 over, the remainder being the *last* figure of the dividend, and the quotient the number formed by the preceding figures; again, $2173 = 2100 + 73 = 100 \times 21 + 73$, therefore, $2173 \div 100 = 21$ and 73 over, where the remainder is made up of the last *two* figures of the dividend, and the quotient of the preceding figures; also, $52173 = 52000 + 173 = 1000 \times 52 + 173$, therefore, $52173 \div 1000 = 52$ and 173 over, the remainder being composed of the last *three* figures of the dividend, and the quotient of the rest; and so on. Hence:

To divide by such numbers as 10, 100, 1000, *&c., remove from the dividend, beginning at the right hand, as many figures as there are ciphers in the divisor; these, as they*

stand, will form the remainder, and the remaining figures of the dividend, the quotient.

68. Divide 726523 by 8000. Since $8000 = 1000 \times 8$, we may divide successively by 1000 and 8.

$$\begin{array}{r|l} 1000 & 726{,}523 \\ \hline 8 & 726 \cdot\cdot 523 \\ \hline & 90 \cdot\cdot 6 \end{array} \Big\} 6523 \qquad\qquad \begin{array}{r|l} 8{,}000 & 726{,}523 \\ \hline & 90 \cdot\cdot 6523 \end{array}$$

Remainder $= 1000 \times 6 + 523 = 6000 + 523 = 6523$, that is, is made up of the remainder arising from dividing 726 by 8, followed by the last three figures of the dividend. Hence, we may work the example in this way: strike from the dividend the last *three* figures (*three* being the number of ciphers at the end of the divisor), divide the preceding part of the dividend by 8 (that part of the divisor left when the ciphers are removed); the quotient is the required quotient; place the remainder (6) before the figures struck off from the dividend (523), and the proper remainder is formed. The work will stand as in the second form.

Next, divide 8727423 by 20300. Since $20300 = 100 \times 203$, we may divide by 100 and 203 successively.

$$\begin{array}{r|l} 100 & 87274{,}23 \\ \hline 203 & 87274 \cdot\cdot 23 \\ \hline & 429 \cdot\cdot 187 \end{array} \Big\} 18723 \qquad\qquad \begin{array}{r} 203{,}00)87274{,}23(429 \\ 812 \\ \hline 607 \\ 406 \\ \hline 2014 \\ 1827 \\ \hline 18723 \end{array}$$

Remainder $= 100 \times 187 + 23 = 18700 + 23 = 18723$, that is, is made up of the remainder from the division of 87274 (the part of the dividend left when the last *two* figures are removed) by 203 (that part of the divisor preceding the *two* ciphers at the end) followed by the two figures cut off from the right of the dividend. Hence the second form.

These examples make clear the following rule:—

If the divisor be terminated by ciphers, remove from the dividend, going from the right hand, as many figures as there are ciphers at the end of the divisor, and from the divisor all these ciphers; divide the remaining part of the dividend by the remaining part of the divisor, and the quotient is the one required; after the remainder write the figures struck from the dividend, and the proper remainder is formed.

This rule includes those cases where the figures removed from the end of the dividend are altogether, or in part, ciphers.

THE END.

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